Several aspects of the interaction physics between an emerging planetary system and its precursor nebula are reviewed. The principal interaction mechanism is through density waves and their associated torques. Two types of orbital migration of protoplanets are distinguished, and the conditions favoring each are delineated. The effects of migration on the style and timescales of planetary formation and on protoplanet survival are discussed.

I. INTRODUCTION

Aside from the primary, the most massive component of a forming planetary system is the precursor circumstellar disk. And yet, the gravitational influence of a disk on an emerging planetary system has been largely ignored throughout most of the cosmogonical literature. The disk's role as a source of planetary material is, of course, obvious at the outset, and its influence on the orbits of small objects through aerodynamic drag has been recognized since the work of Whipple (1972) and Weidenschilling (1977), but direct gravitational effects were not considered until almost a decade later. Even then, an appreciation of the many aspects of disk-planet gravitational coupling has only slowly worked its way into cosmogonical modeling, over the last fifteen years or so. Lately, however, this process has accelerated because of the discovery of extrasolar planets, particularly those in tight orbits around their primaries. Given the apparent difficulty of forming planets of a jovian mass or more in close stellar proximity, their existence seems to constitute strong circumstantial evidence of large-scale migration of planetary objects, a suggestion that seemed almost heretical a few years ago. The angular momentum change associated with large-scale migration is considerable. Except for the possibility of mutual scattering between giant planets, only the disk provides a large enough angular momentum reservoir to effect such modifications.

The first study of collective gravitational interaction with a disk in a planetary context was the proposal by Goldreich and Tremaine (1978) that the Cassini division in Saturn's ring system could have been opened and be maintained by Mimas through its 2:1 mean-motion resonance. This is a Lindblad resonance, in which the forcing frequency, seen from a reference frame moving with disk material, matches the disk's natural oscillation frequency, which is (nearly) the local epicycle frequency κ (e.g., Lynden-Bell and Kalnajs 1972). The calculations employed a fluid-mechanical description of a self-gravitating ring that demonstrated the importance of wave action in redistributing the (negative) angular momentum deposited at resonance to nearby portions of the ring, thereby accounting for the large width of the Cassini division compared to the nominal width of the resonance. In a closely related problem, Lin and Papaloizou (1979) utilized an impulse model of disk-planet interaction to show that a large enough secondary could truncate a circumstellar disk at a low-order Lindblad site.

Modification of the perturber's orbit by disk torques was first addressed fully by Goldreich and Tremaine (1980). The resonant forcing of the disk results in the launching of a spiral density wave train (e.g., Goldreich and Tremaine 1979; Papaloizou and Lin 1984; Ward 1986; Lin and Papaloizou 1993; Artymowicz, 1996*b*). The attraction of the perturber for this nonaxisymmetric surface density, σ, results in the torque, sometimes collectively referred to as disk *tidal* torques in analogy to solid-body tides between a planet and its satellite. In general, the interactions at various mean-motion resonances result in *work* being done on the planet as well as angular momentum exchange. From the Jacobi constant, the work done on the planet is $\dot{E} = \Omega_{\rm ps}\dot{L}$, where $\Omega_{\rm ps}$ represents the pattern speed of the resonant term, and where

$$E = \frac{-GM_p M_\star}{2a_p} \qquad L = M_p\sqrt{GM_\star a_p(1-e_p^2)} \tag{1}$$

are the energy and angular momentum of the planet, respectively (Goldreich and Tremaine 1980). This leads to equations for the rates of change of the planet's a_p and e_p due to resonant torque $T_{l,m}$;

$$\frac{\dot{a}_p}{a_p} = \frac{2T_{l,m}}{M_p a_p^2 \Omega_p}\frac{\Omega_{\rm ps}}{\Omega_p} \qquad \frac{\dot{e}_p}{e_p} = \frac{-T_{l,m}(1-e_p^2)^{1/2}}{e_p^2 M_p a_p^2 \Omega_p}\left(1 - \frac{\Omega_{\rm ps}}{\Omega_p}\sqrt{1-e_p^2}\right) \tag{2}$$

with Ω_p denoting the planet's mean motion.

II. DISK TORQUES

Each $\{m, l\}$ term in the planet's Fourier-expanded disturbing potential

$$\begin{aligned}\phi &= -\frac{GM_p}{|\mathbf{r}-\mathbf{r}_p|} + \frac{M_p}{M_\star}\Omega^2(\mathbf{r}\cdot\mathbf{r}_p) \\ &= \sum_{l=-\infty}^{\infty}\sum_{m=0}^{\infty}\phi_{l,m}\cos[m(\theta-\Omega_p t)-(l-m)\kappa_p t]\end{aligned} \tag{3}$$

has a pattern speed $\Omega_{\rm ps} \equiv \Omega_p + (l - m)\kappa_p/m$ and gives rise to a corotation resonance (CR, $\Omega = \Omega_{\rm ps}$) plus an inner [ILR, $m\left(\Omega - \Omega_{\rm ps}\right) = \kappa$] and an outer [OLR, $m\left(\Omega - \Omega_{\rm ps}\right) = -\kappa$] Lindblad resonance. In most applications, the disk's epicycle frequency is nearly its angular velocity $\Omega(r)$. The wave train launched from each Lindblad resonance carries an angular momentum flux (e.g., Goldreich and Tremaine 1979)

$$\mathcal{F} = -\mathrm{sgn}(k)\frac{mr\Phi^2}{4G}\left[1 - \frac{c^2|k|}{\pi G\sigma}\right] \tag{4}$$

where Φ is the portion of the disk's gravitational potential associated with the wave, c is the gas sound speed, and k is the wavenumber that satisfies the dispersion relation

$$m^2\left(\Omega - \Omega_{\rm ps}\right)^2 = \kappa^2 - 2\pi G\sigma|k| + c^2k^2 \tag{5}$$

We have used the sign convention of Goldreich and Tremaine (1979), where the waves are trailing (leading) for $k > 0$ ($k < 0$), and the group velocity of the waves is given by

$$c_g = \mathrm{sgn}(k)\left(\frac{\pi G\sigma - c^2|k|}{m(\Omega - \Omega_{\rm ps})}\right) \tag{6}$$

Therefore, the angular momentum density of the waves as defined by $2\pi rc_g\mathcal{H} = \mathcal{F}$, is (e.g., Goldreich and Tremaine 1978)

$$\mathcal{H} = -\left(\frac{\Omega - \Omega_{\rm ps}}{2\sigma}\right)\left(\frac{m\Phi}{2\pi G}\right)^2 \tag{7}$$

and has the same sign as the torque on the disk. OLRs exert a positive torque, whereas ILRs exert a negative torque. The corresponding reaction torques $T_{l,m}$ on the planet are of opposite sign.

The torque exerted on an object by the disk due to interaction at a Lindblad resonance is (Goldreich and Tremaine 1979),

$$T^L_{l,m} = f_L(\xi)\mathrm{sgn}\left(\Omega - \Omega_{\rm ps}\right)\frac{\pi^2\sigma\Psi^2_{l,m}}{|rdD/dr|} \tag{8}$$

where $\Psi_{l,m} \equiv rd\Phi_{l,m}/dr + 2\Omega\Phi_{l,m}/\left(\Omega - \Omega_{\rm ps}\right)$ is the forcing function, and the so-called cutoff function $f_L(\xi)$ has been introduced, with $\xi \equiv mc/r\Omega$. The cutoff function approaches unity in a cold disk ($c = 0$) but kills the torque at high ξ. The high-ξ behavior is due to a pressure-induced shift of the resonance sites away from the perturber when the azimuthal wavelength $\sim r/m$ of the forcing term is less than the scale height $\sim c/\Omega$ of the disk (Ward 1988, 1997*a*; Artymowicz 1993*b*). The quantity D that appears in the denominator is the frequency distance,

$$D \equiv \kappa^2 - m^2\left(\Omega - \Omega_{\rm ps}\right)^2 \tag{9}$$

which vanishes at the Lindblad resonance. From the dispersion relationship [equation (5)], *long* waves for which $\pi G\sigma > c^2|k|$ must propagate where $D > 0$. Causality requires that the group velocity be directed away from the resonance, or $\mathrm{sgn}(c_g) = \mathrm{sgn}(dD/dr)$ when evaluated at the resonance. For $m \geq 2$, $\mathrm{sgn}(dD/dr) = \mathrm{sgn}\left(\Omega - \Omega_{\mathrm{ps}}\right)$, and comparing this with equation (6) reveals that the waves are trailing, $k > 0$. The $m = 1$ resonances are a special case; the OLR waves are trailing, but for the ILR the waves are launched from a secular resonance where the apsidal precession rate $d\tilde{\varpi}/dt = g(r)$ of the disk matches that of the planet; i.e., $g(r) = g_p$ (Ward and Hahn 1998*a,b*). For this case, $\mathrm{sgn}(k) = \mathrm{sgn}(dg/dr)$, and waves can be either leading or trailing depending on the gradient of $g(r)$.

Finally, the torque on the perturber due to a corotation resonance is (Goldreich and Tremaine 1979, 1980)

$$T_{l,m}^{C} = f_C(\xi)\frac{m\pi^2\Phi_{l,m}}{-d\Omega/dr}\frac{d}{dr}\left(\frac{\sigma}{B}\right) \tag{10}$$

where $B \equiv (2r)^{-1}d\left(r^2\Omega\right)/dr$ is the vorticity parameter and f_c is the cutoff function for this type of resonance (e.g., Ward 1989). For a review of many of these particulars the reader is directed to the chapters on this subject in the earlier volumes of the *Protostars and Planets* series, notably Lin and Papaloizou (1985, 1993).

III. PROTOPLANET MIGRATION

A. Type I: Dynamical Friction

The most important resonances for changing the semimajor axes are those associated with terms in the Fourier-expanded disturbing function that are zero-order in eccentricity. From equations (2) and (8) we see that the inner resonances expand the orbit, whereas the outer ones contract it. Unless these various torques all exactly cancel, a net torque ΔT on the planet is expected and, with it, possible orbital evolution of the perturbing body. Goldreich and Tremaine (1980) pointed out that a mismatch in the cumulative inner and outer tidal torques was likely and reasoned that the fractional torque difference should be of order h/r, where $h \sim c/\Omega$ is the vertical scale height of a gas disk. They refrained from predicting the sign, cautioning that this was hard to ascertain without detailed knowledge of the disk. Ward (1986) argued that this is overly pessimistic and showed that the near-Keplerian rotation results in a negative torque bias ΔT for a wide range of disk models, a result since confirmed by the numerical integrations of Korycansky and Pollack (1993). The resulting orbital decay rate is

$$\dot{r} = \frac{2\Delta T}{M_p r\Omega} \approx c_{\mathrm{I}}\mu\left(\frac{\sigma r^2}{M_\star}\right)\left(\frac{r\Omega}{c}\right)^3 r\Omega \tag{11}$$

where $\mu \equiv M_p/M_\star$, and $c_\mathrm{I} < 0$ is an asymmetry constant proportional to $c/r\Omega$. In earlier discussions of this problem (Ward 1997*a*, 1998*b*), c_I of order unity was assumed, which is appropriate for a nebula scale height about $h \sim 10^{-1}r$. In order to reveal the functionality better, here we use the relatively constant parameter $C_a \equiv |c_\mathrm{I}|(r\Omega/c)$ introduced by Ward (1986). Note that this implies $\dot{r}$ itself scales as $(r\Omega/c)^2$, as described by Goldreich and Tremaine (1980). We designate this sort of behavior as *Type I*. Both analytic and numerical calculations indicate that $C_a \approx O(1\text{–}10)$; e.g., Ward (1986, 1997*a*); Korycansky and Pollack (1993), Takeuchi and Miyama (1998). The orbital drift of a planet in a minimum-mass solar nebula has a characteristic decay time $\tau_\mathrm{I} \sim |r/\dot{r}|$ of

$$\tau_\mathrm{I} = \frac{\Omega^{-1}}{C_a}\left(\frac{M_\star}{M_p}\right)\left(\frac{M_\star}{\sigma r^2}\right)\left(\frac{c}{r\Omega}\right)^2 \tag{12}$$

B. Type II: Coevolution

Papaloizou and Lin (1984) and Lin and Papaloizou (1985, 1993) concentrated on the reaction of the disk, including the case where a planet truncates the disk both inside and outside its orbit, thereby creating a gap in which it then resides. The ability of a planet to open and maintain a gap depends in part on the viscosity ν of the disk (Hourigan and Ward 1984; Papaloizou and Lin 1984, 1993; Ward and Hourigan 1989) as well as the planet's mass and the damping characteristics of the waves (Takeuchi et al. 1996). When conditions are met, the gap presents a barrier to any radial flow of disk material that may occur due to global viscous diffusion. The planet in effect locks itself into the angular momentum transport process of the disk by acting as a sort of "relay station" that transmits angular momentum across the gap via tidal torques. As the disk evolves, the planet will maintain its *relative* position to the disk material. For this to happen, the disk must adjust its local configuration so that the torque differential ΔT is just that required for the planet to drift at the same rate, $\sim O(\nu/r)$, as the gas. For a Sakura-Sunyaev viscosity prescription, the planet's migration rate is

$$\dot{r} \approx c_\mathrm{II}\alpha\left(\frac{c}{r\Omega}\right)^2 r\Omega \tag{13}$$

where c_II is also a constant of order unity. This is the migration type proposed as a delivery mechanism for emplacement of close stellar companions such as 51 Pegasi b (Lin et al. 1996). Trilling et al. (1998) have presented simulations of this behavior. We designate this sort of migration as *Type II*.

Although both types of orbital drift are described in the literature, they have not always been clearly distinguished from one another. In fact, these cases represent the weak and strong coupling limits of a range of disk-planet interactions (Ward 1997*a,b*), and type-casting these distinctly

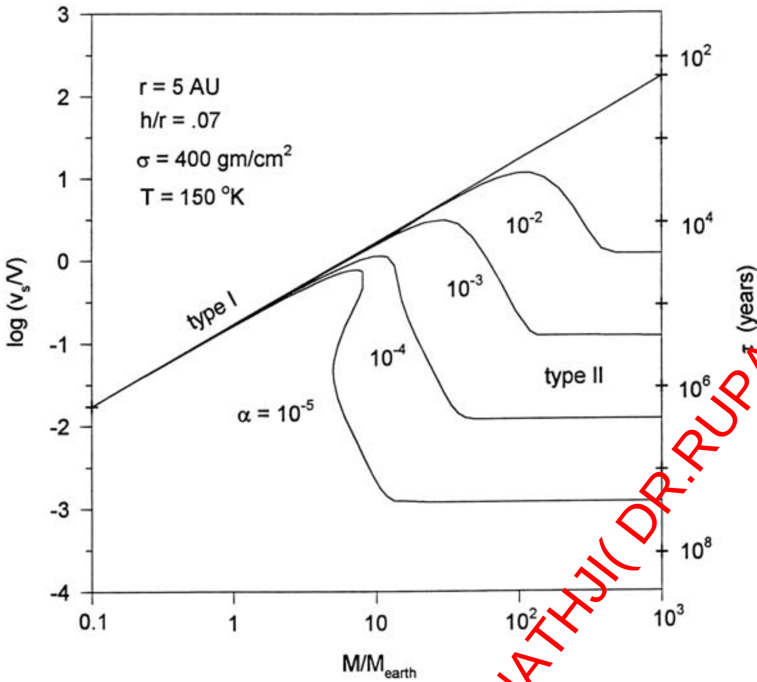

Figure 1. Characteristic decay times $\tau \equiv r/\dot{r}$ (right-hand scale) as a function of protoplanet mass, measured in Earth masses. Curves are labeled by the viscosity parameter $\alpha \equiv \nu/c^2\Omega^{-1}$ and constructed for a minimum-mass solar nebula at a distance of 5 AU. The timescale decreases inversely with mass (Type I) until it reaches a threshold size M_S (Shiva limit), past which the time scale increases (Type II) as a gap progressively opens. When a gap is fully established, the planet coevolves with the disk with a timescale inversely proportional to α. The right-hand scale gives the migration rate normalized to $V \equiv 2\mu_\oplus\mu_d(r/h)^3 r\Omega$, which is comparable to the Type I drift velocity for an Earth-sized object. Taken from Ward (1997*a*).

different behaviors proves useful in clarifying discussions. For a given disk, there is a critical mass where a transition from Type I to Type II occurs (Ward 1997*a*). Figure 1 shows the radial drift velocity as a function of protoplanet mass evaluated at ~5 AU in a minimum-mass solar nebula. The different curves are labeled by the assumed value of α. The velocity drops by as much as two orders of magnitude at transition.

C. Gap Formation

The type of behavior executed by a protoplanet depends on whether it can open and maintain a gap. A gap can be maintained against viscous diffusion if the cumulative torque $T_o \sim O[\mu^2(\sigma r^2)(r\Omega)^2(r/h)^3]$ that the planet exerts on one side of the disk exceeds the viscous couple $g \approx 3\pi\nu\sigma r^2\Omega$

(e.g., Lin and Papaloizou 1985; Ward 1986). For this to be satisfied, the protoplanet mass must exceed

$$\mu_\nu \sim c_\nu \left(\frac{\nu}{r^2\Omega}\right)^{1/2} \left(\frac{c}{r\Omega}\right)^{3/2} \sim c_\nu \sqrt{\alpha} \left(\frac{h}{r}\right)^{5/2} \tag{14}$$

where $c_\nu \sim O(1-10)$. Adopting a scale height $h \sim c/\Omega \sim 0.07r$, this mass limit becomes $\mu_\nu \sim c_\nu \sqrt{\alpha} \times 10^{-3}$. Lin and Papaloizou (1985) suggest that gap development terminates a giant planet's gas accretion phase and sets an upper mass limit for protoplanets. If Jupiter ($\mu_{\rm Jup} = 10^{-3}$) represents such a limit, a turbulence parameter of $\alpha \sim$ few $\times 10^{-2}$ is inferred. However, a much smaller value, $\alpha \sim 10^{-4}$–10^{-3}, is usually quoted from attempts to estimate nebula turbulence (e.g., Cabot et al. 1987; Dubrulle 1993). Note also that the Type II decay time for the higher value of α is much shorter than the probable lifetime of disks inferred from observation. How do we reconcile this with the existence of Jupiter and the numerous extrasolar planets of comparable size or larger that have been recently discovered?

Lin and Papaloizou (1993) assert that in addition to a sufficiently strong torque, it is necessary that the protoplanet's Hill radius, $R_H = r(\mu/3)^{1/3}$ exceed the scale height of the disk. This would set a lower limit for gap formation of $\mu_h \sim 3(h/r)^3 \sim O(10^{-3})$ even if α is small. The justification given for this condition is that a gap narrower than $O(h)$ would be Rayleigh unstable, which seems to implicitly assume that all objects would tend to open gaps of width comparable to their respective Hill radii. In fact, the scale height sets the minimum gap width for objects smaller than μ_h because pressure effects prevent high-order Lindblad resonances from lying closer than that to the protoplanet (Artymowicz 1993*a*; Ward 1988, 1997*a*). Accordingly, the validity of this so-called "necessary" condition is suspect.

On the other hand, the ability of a planet to open a gap also depends on the propagation distance of the waves, because the angular momentum density of the waves is not permanently transferred to the disk until the waves damp; before that happens, the disk cannot undergo secular adjustment in structure. If one assumes that the damping length $\ell \gtrsim h$ of the waves better represents the minimum gap width that can be opened, then (provided the gap is not so large as to exclude all resonances) the torque exerted on a disk with an edge distance ℓ is down by a factor $\sim(\ell/h)^{-3}$ from T_o, and the planet mass needed to maintain the gap increases to $\mu_{\nu,\ell} \sim c_\nu \sqrt{\alpha}(h/r)^{5/2}(\ell/h)^{3/2}$. This still leaves a quandary: If gap formation is delayed, what keeps the planet from spiraling into the primary via Type I migration? Perhaps the solution is the onset of gas accretion. If accretion can deplete the local surface density of the disk sufficiently to override the intrinsic torque asymmetry, but not so much as to shut off accretion, a survival strategy might be found. Models of gas accretion by a migrating planet have not yet been devised but should be a high-priority problem.

D. Wave Damping

Several processes can damp density waves:

(i) *Viscosity*. Goldreich and Tremaine (1980) and Ward (1986) both considered damping of waves in a high-Q disk by turbulent viscosity. In a more thorough treatment, Takeuchi et al. (1996) find that viscosity can dissipate the waves over a distance $\ell_\nu \sim \alpha^{-2/5}h$. In this case the critical mass is $\mu_{\nu,\ell} = 10^{-3}c_\nu\alpha^{-1/10}$, which, considering our order of magnitude approach, is still comparable to the masses of extrasolar planets for $\alpha < 10^{-2}$. However, other processes may supersede viscous damping for small α.

(ii) *Nonlinearity*. If $\mu \gtrsim (h/r)^3$, the perturbed surface density achieves nonlinear status locally and may shock-dissipate relatively close ($\sim h$) to the planet (e.g., Ward 1986). Indeed, this seems a more likely cause of gap formation when an object's Hill radius exceeds h, rather than the above-mentioned Rayleigh argument.

Other promising damping mechanisms include (iii) *radiation damping* (Cassen and Woolum 1996), in which the temperature rise during wave compression increases the radiation rate from the surface of the disk, and (iv) *channeling* (Lubow and Ogilvy 1998), a process where the wave energy becomes concentrated near the surface of the disk in a layer of thickness $\sim 1/k$, with k being the wavenumber. This occurs when there is a vertical temperature gradient. As the wave propagates, the wavenumber increases, confining the energy to a progressively thinner layer until nonlinearities set in.

Both of these latter mechanisms would appear to work even if α is very low. Indeed, recent work by Stone and coworkers (1996) claims that vertical convection in the disk (the principally invoked cause of turbulence) does not couple well to the horizontal motions, as has usually been assumed, and will not significantly contribute to the viscous evolution of the disk. This has led Gammie (1996) to suggest a layered model of the solar nebula, where turbulence is generated by magnetohydrodynamic (MHD) mechanisms but is confined to the surface, where cosmic rays can maintain a requisite degree of ionization. Coupled with channeling, this suggestion indicates that it may be possible to damp waves fairly locally in a disk with a very low effective α.

Even in an inviscid disk, however, there is a lower limit to the mass of an object that can open a gap, because of inertial effects. If the planet drifts across the gap in less time than it can excavate it, the gap will fail to open (Hourigan and Ward 1984; Ward and Hourigan 1989; Takeuchi et al. 1996). This so-called "inertial" limit is the inviscid limit of the Shiva mass (see below) and is typically of order

$$\mu_i = \frac{C_a^2}{4\pi}\left(\frac{\pi\sigma r^2}{M_\star}\right)\left(\frac{c}{r\Omega}\right)^3 \approx 8\mu_d\left(\frac{h}{r}\right)^3 \tag{15}$$

where $\mu_d \equiv \pi\sigma r^2/M_\star$ is a normalized measure of the disk mass. For $\mu_d \sim$ few $\times 10^{-3}$, $h/r = 0.07$, the inertial mass is a few Earth masses.

E. Torque at Corotation

So far we have paid little attention to the $m = l$ corotation resonances that fall at the perturber's semimajor axis. The disk's response to this forcing is to execute horseshoe-type behavior (see chapter by Lin et al., this volume). Horseshoe streamlines in an annulus of half-width w result in a torque on the perturber, of the form (Ward 1991)

$$T_{hs} = 4|A|B^2 w^4 r \frac{d}{dr}\left(\frac{\sigma}{B}\right) \tag{16}$$

where $A \equiv (r/2)d\Omega/dr$; $B \equiv (2r)^{-1} d\left(r^2\Omega\right)/dr$ are the Oort parameters of the disk. As one should expect, this is the same functional dependence on the vortensity as equation (10). For a Keplerian flow, $B = \Omega/4$, implying that T vanishes if $\sigma \propto r^{-3/2}$. Korycansky and Pollack (1993) have numerically integrated the linear response of a disk to an mth-order term and determined which portion of the torque is due to corotation by subtracting the downstream angular momentum flux of the waves from the whole torque. This exploits the fact that only angular momentum deposited at the inner and outer Lindblad resonances will be carried away by wave action. The residual corotation torques for each m are then summed to find the total at corotation, obtaining the approximate result

$$T_C \approx \frac{4}{3}\mu^2 \left(r\Omega\right)^2 \left(\frac{r\Omega}{c}\right)^2 \left[r^3 B \frac{d}{dr}\left(\frac{\sigma}{B}\right)\right] \tag{17}$$

This result can also be obtained from equation (16) if one assumes that the particles have an eccentricity $e \sim h/a$ and that the horseshoe streamline coming closest to the perturber is turned around at a distance $\sim 2ea$ equal to half the axis of the epicycle in the azimuthal direction. The ratio $T_C/T_o \sim 4h/3r \ll c_{\rm I}$ implies that it is unlikely that the corotation torque could balance out the differential Lindblad torque. Furthermore, the corotation torque is subject to saturation on the horseshoe libration timescale $t_{\rm sat} \sim (r/w)\Omega^{-1}$ and so could, at best, reduce the drift rate to $\dot{r} \sim O[\mu(r/h)r\Omega]$, unless turbulence prevents saturation. This could be a limiting factor only if $\mu_d > O(10^{-2})$.

The above discussion notwithstanding, the corotation zone is among the least well-understood resonant regions. Lin et al. (this volume) point out that there are complex nonlinear effects, especially on the scale of the protoplanet's Hill radius, $R_H = r(\mu/3)^{1/3}$, that manifest themselves in recent numerical simulations of the flow. They suggest that these features may be important contributors to the protoplanet's orbital evolution, perhaps countering the torques inferred from linear analyses. We await such a possible revelation with interest, because current estimates of dynamical friction seem almost too powerful, and as yet no device other than gap formation or the onset of gas accretion seems capable of short-circuiting it.

IV. ACCRETION

Ward and coworkers have (e.g., Hourigan and Ward 1984; Ward 1989*b*, 1993*a*) stressed that a source of orbit drift for large objects would constitute an as yet unexploited degree of radial mobility for accretion models. Here we discuss a number of ways in which disk tides could modify our understanding of the planet building process.

A. Runaway Limit

One of the tightest constraints on models of solar system formation is the suspected 10^{6-7} year lifetime of the nebula, inferred from observation of T Tauri stars (e.g., Walter 1986; Walter et al. 1988). Obviously, the existence of gas giants like Jupiter and Saturn establish that the planet-building process for these objects was essentially completed before nebula dispersal. These planets are believed to have acquired their H/He component by gas accretion onto preexisting solid cores with estimated masses of 10–20 $M_\oplus$ (Mizuno et al. 1978; Bodenheimer and Pollack 1986; Podolak et al. 1993). If this model is correct, we must account for the accretion of 10^{29}-g cores within the lifetime of the gas disk.

Current models of solid body accretion indicate that large embryos can form in a relatively short timescale as a result of the onset of accretion runaway (Greenberg et al. 1978; Wetherill and Stewart 1989, 1993; Ida and Makino 1993). This runaway is due to a strong feedback loop in the growth rate, $\dot{M} \sim \sigma_d \Omega \pi R^2 F_g$, through the gravitational enhancement factor F_g where σ_d is the surface density of accretable material in the disk and M and R are the embryo's mass and radius, respectively (e.g., Greenzweig and Lissauer 1993; Lissauer and Stewart 1993). The implied characteristic growth time $\tau_{\text{growth}} \equiv M/\dot{M}$ is

$$\tau_{\text{growth}} \approx \Omega^{-1}\left(\frac{\rho_p R}{\sigma_d}\right)F_g^{-1} \tag{18}$$

where ρ_p is the body density of the embryo. The enhancement factor is the ratio of the effective collision cross section to the geometrical cross section. If the relative velocities v are dominated by velocity dispersion instead of disk shear, the enhancement factor becomes $F_g = 1 + (v_e/v)^2$ where $v_e \equiv (2GM/R)^{1/2}$ is the embryo's escape velocity. When the smaller field particles establish their velocity dispersion via equilibrium between mutual gravitational scattering and inelastic collisions, v will be comparable to the escape velocity, v_e', of a typical field particle, R'. Since the escape velocity of any object is proportional to its radius, $F_g \sim (v_e/v_e')^2 \sim (R/R')^2$ for $R \gg R'$, and the characteristic growth time for the runaway is inversely proportional to R. There is a limit, however, to the size of F_g. At some point the embryo begins to stir the surrounding swarm of particles and to contribute to their dispersion velocities (Lissauer 1987). In this case the enhancement factor approaches a limiting value of order

$F_g \rightarrow O(R_H/R)$, where R_H is the protoplanet's Hill radius. This ratio is dependent only on heliocentric distance and protoplanet density and, for a solar-mass star, is $\sim 133(\rho_p/1\text{g/cm}^{-3})^{1/3}(r/\text{AU})$. At 5 AU, a 3-g/cm^3 protoplanet would have an enhancement factor of order $\sim 10^3$. Once this limit is reached, the growth timescale becomes proportional to R [see, e.g., Lissauer and Stewart (1993) and Ward (1996) for recent readable reviews of solid body accretion].

This runaway phase has been generally thought to stall down by isolation of the embryo; that is, when it has cannibalized the disk locally, it simply runs out of material to sustain its growth (e.g., Wetherill 1990; Lissauer and Stewart 1993). Further accretion would then seem to depend on long-range gravitational interactions with other embryos, which will eventually generate crossing orbits. In this case, the enhancement factor reverts to a value of order unity, and the growth timescale lengthens accordingly. The so-called runaway mass limit is

$$M_{\text{run}} \equiv 3^{1/4}\left(\frac{8\pi\sigma_d r^2}{M_\star}\right)^{3/2} M_\star \tag{19}$$

which, for a minimum model of the solar nebula, is well short of a giant planet core. Binary accretion with $F_g \sim O(1)$ is too slow to form cores within the probable $\sim 10^{6-7}$ year duration of the nebula. However, it now seems that this concern was probably unfounded, because disk tides will prevent isolation by causing the embryo to migrate into undepleted regions of the disk (Ward and Hahn 1995; Tanaka and Ida 1998). The migrating embryo sweeps past fresh material equal to its own mass in a time $\tau_{\text{swp}} \equiv (M_p/2\pi\sigma_d r^2)\tau_{\text{I}}$. If this is less than τ_{growth}, accretion continues unabated.

B. Dispersion Velocities

For the eccentricity of a perturber, the most important potential terms are those that are first-order in e, i.e., those with $l = m \pm 1$ (Goldreich and Tremaine 1980). From equation (1), the sign of the eccentricity change is equal to $\text{sgn}(T_{l,m}) \times \text{sgn}(\Omega_{\text{ps}}\sqrt{1-e^2} - \Omega_p)$. For Lindblad resonances, $\text{sgn}(T_{l,m}) = \text{sgn}(\Omega - \Omega_{\text{ps}})$, so that inner (outer) resonances exert positive (negative) torques on the perturber. With the faster pattern speed, $\Omega_{\text{ps}} = \Omega_p + \kappa_p/m$, negative torques from outer Lindblad resonances damp the eccentricity while positive torques from inner resonances excite it. Just the reverse is true for the slower pattern speed, $\Omega_{\text{ps}} = \Omega_p - \kappa_p/m$. Consequently, the eccentricity is excited by those resonances that fall well inside and outside of the orbit (external) but damped by those that fall in the vicinity of the perturber's orbit (coorbiting). These behaviors are summarized in Table I.

For corotation resonances, $\text{sgn}(T_{l,m}) = \text{sgn}(d(\sigma/B)/dr)$. If there is no gap and the gradient in the vortensity, σ/B, does not switch sign across the orbit, the fast and slow corotation resonances oppose each other and

TABLE I
Zero- and First-Order Resonances: Locations and Signs

	Radius				
Ω_{ps}	$a\left(1-\frac{4}{3m}\right)$	$a\left(1-\frac{2}{3m}\right)$	a	$a\left(1+\frac{2}{3m}\right)$	$a\left(1+\frac{4}{3m}\right)$
$\Omega_p+\frac{\kappa_p}{m}$ $\phi \propto e^1$	ILR $\dot{e}>0$	CR $\dot{e}=\pm$	OLR $\dot{e}<0$		
Ω_p $\phi \propto e^0$		ILR $\dot{a}>0$	CR $\dot{a}=\pm$	OLR $\dot{a}<0$	
$\Omega_p-\frac{\kappa_p}{m}$ $\phi \propto e^1$			ILR $\dot{e}<0$	CR $\dot{e}=\mp$	OLR $\dot{e}>0$

Zero-order terms are the most important for semimajor axes, while eccentricities are most influenced by terms proportional to e. The former have pattern speeds Ω_{ps} equal to the planet's mean motion Ω_p; the latter have pattern speeds either slightly faster or slower than Ω_p by an increment κ_p/m. Each Fourier term has a corotation and two Lindblad resonances. The resonances due to the fast and slow terms have their sites shifted inward and outward respectively from the zero-order terms. The signs of da/dt and de/dt due to Lindblad resonances depend on the frequencies involved, as explained in the text. For corotation torques, the sign depends on the gradient of σ/B. Adapted from a figure by Goldreich and Tremaine (1980).

largely cancel out (Ward 1988). However, if the protoplanet occupies a gap, $d(\sigma/B)/dr$ will be negative (positive) inside (outside) the orbit, so *both* corotation resonances damp the eccentricity (Goldreich and Tremaine 1980). Also, in this case, the coorbiting Lindblad resonances shut off, because they fall within the gap. Goldreich and Tremaine (1980) compared the excitation rate of external Lindblad resonances to the damping rate of corotation resonances caused by a ring of material and concluded that the latter dominated slightly; that is, $\left|\dot{e}/e\right|_L / \left|\dot{e}/e\right|_C = 0.95$. However, this result assumes that there is no saturation among the resonances. Indeed, the stability of the orbit of a gap-confined protoplanet against eccentricity growth is still not well determined.

Recently, Ward and Hahn (1998*a,b*) have shown that strong eccentricity damping accompanies the $m = 1$, ILR of the slow-pattern-speed term. The pattern speed is $\Omega_{ps} = \Omega_p - \kappa_p \equiv d\tilde{\omega}_p/dt$, which is the precession rate of the planet's longitude of perihelion, $\tilde{\omega}_p$. The pattern's slow rotation rate results in a very long wavelength for the one-arm spiral wave (called an apsidal wave), which, in turn, allows for inordinately strong coupling to the planet's gravitational potential. The damping rate for this resonance is given by

$$\frac{1}{e^2}\frac{de^2}{dt} = -\frac{\pi}{4}\beta^{3/2}\left[b_{3/2}^{(2)}(\beta)\right]^2 \mu_p \mu_d \left(\frac{\Omega}{\left|r\, dg(r)/dr\right|}\right)\Omega_p \tag{20}$$

where $\beta \equiv r_{res}/a_p$. This is a factor of order $\sim \Omega_p/\left|g_p\right| \gg 1$ greater than other low-order Lindblad resonances for $m \gtrsim 2$. Furthermore, since the location of a secular resonance is sensitive to the mass of the protoplanet and the disk, it may fall within the disk even if the planet occupies a gap. This situation may have application to the orbital circularization of extrasolar planets (Ward 1998*a*). Tremaine (1998) has since derived equation (20) in his discussion of "resonant friction" in a planetesimal disk. We prefer the name "secular resonant damping" (SRD), because the term "friction" usually implies energy dissipation, whereas very little energy [specifically, $\dot{E} = (d\tilde{\omega}_p/dt)L$] is carried by apsidal waves launched from a secular resonance.

Returning to the case where no gap exists, the situation is more transparent. The strength of the coorbiting Lindblad resonances exceeds that of external resonances by a comfortable margin (Ward 1988) as shown in Fig. 2. Consequently, the eccentricity decays with a characteristic timescale of

$$\tau_e = \frac{\Omega^{-1}}{C_e}\left(\frac{M_\star}{M_p}\right)\left(\frac{M_\star}{\sigma r^2}\right)\left(\frac{c}{r\Omega}\right)^4 \tag{21}$$

The constant C_e depends on the disk model but is of order unity (Artymowicz 1993*a*). Note that the stronger dependence on $c/r\Omega$ makes this timescale much shorter than the migration time of equation (12). For large objects, this timescale can be short compared to gas drag or collisional damping (Ward 1993*a*). Figure 3 shows the equilibrium velocities found by equating various damping rates to the gravitational relaxation rate of

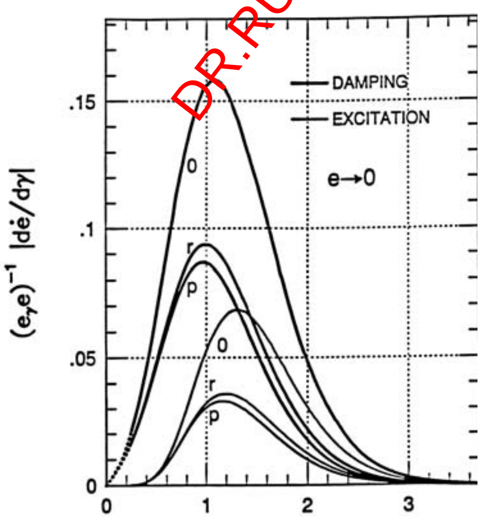

Figure 2. Comparison of the damping rate from coorbiting Lindblad resonances to the excitation rate due to external Lindblad resonances. The parameter, $\gamma \equiv 2(mc/r\Omega)^{2/3}/3^{1/3}$. The three pairs of curves are labeled by different treatments of the disk thickness. "o" is a 2D disk, "r" is a vertically averaged response, and "p" is a vertically averaged potential. In each case, the damping strength comfortably exceeds excitation. Taken from Artymowicz (1996*b*).

a disk composed of equal-mass objects (Ward 1993*a*). The masses are normalized to a so-called gravitational mass, M_G, which is the mass that has an escape velocity equal to the gas sound speed. The calculations are performed in a 150-K minimum-mass nebula at 5 AU, for which $M_G = 1.5 \times 10^{25}$ g, or about a fifth of a lunar mass. The disk tidal damping begins to dominate over a widening range of eccentricities as M rises above M_G and tends to limit the dispersion velocities to the sound speed c. This keeps the planetesimal disk flatter, which, in turn, shortens the collision timescale. At first blush, this would seem to eliminate the accretion timescale problem (that it takes too long to form the giant planet cores by the conventional accretion model). A little reflection, however, reveals that things are not so simple. When objects achieve Earth size, the eccentricities are so strongly damped that orbits become noncrossing and collisions cease. Close encounters no longer produce strong scattering events, which explains why the equilibrium velocity drops precipitously in Fig. 3. In this situation, further accretion would have to await disk-induced changes in the planetesimals' semimajor axes.

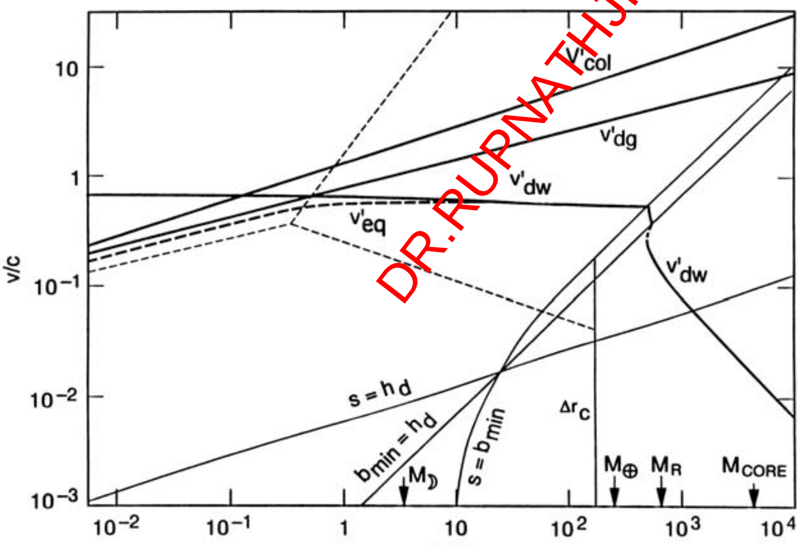

Figure 3. Equilibrium velocities in a unimodal disk as a function of mass for three different sources of damping. The mass is normalized to the value M_G for which the escape velocity equals the sound speed; velocities are normalized to the sound speed c. The short dashed lines partition the diagram into regions dominated by each damping mechanism. They are clockwise from the lower left: collisions, gas drag, and disk torques. Disk torques keep velocities mildly subsonic until object spacing renders the disk collisionless (indicated by Δr_c); then velocities drop quickly with mass. Also shown are boundaries where various combinations of disk thickness h_d, accretion radius S, and minimum impact parameter $b_{\min}$, are equal. Taken from Ward (1993*a*).

C. Nonisotropic Dispersion

In a real disk, where there is a range of object sizes, there will be differential decay in semimajor axes that can force further interactions. Recall that the time it takes a protoplanet to sweep past an equal mass of smaller, nearly stationary solid debris of surface density σ_d is

$$\tau_{\rm swp} = \frac{\Omega^{-1}}{2\pi f_d C_a}\left(\frac{M_\star}{\sigma r^2}\right)^2\left(\frac{c}{r\Omega}\right)^2 \qquad (22)$$

which is independent of protoplanet mass (Ward 1986). Here, $f_d = \sigma_d/\sigma$ represents the solid-to-gas ratio of the nebula. The ratio $\tau_{\rm swp}/\tau_{\rm growth} \equiv \mathscr{E}$ represents an accretion efficiency (Tanaka and Ida 1998). In terms of the "optical depth," $\tau_p = \sigma_d/\rho_p R_p$, of embryos, $\mathscr{E} = (\tau_p F_g/2\pi f_d C_a) \times (M_\star/\sigma r^2)^2(c/r\Omega)^2$ unless this expression exceeds unity, in which case $\mathscr{E} = 1$.

In the most general terms, the accretion rate can be written as $\dot{M} \sim A \times (\rho v)$, where A is an effective collision cross section and ρv represents a mass flux. The spatial density of accretable material is of order $\rho \sim \sigma_d/h_d$, where h_d is the scale height of the particle disk. The effective collision radius due to gravitational focusing is $S \sim R\sqrt{1 + v_{\rm esc}^2/v^2}$. We can define an accretable column density as $\tilde{\sigma} \equiv \rho S \sim \sigma_d(S/h_d)$ unless $S > h_d$, in which case $\tilde{\sigma} \to \sigma_d$. Considering the situation where the particle dispersion velocity v is much less than the protoplanet's escape velocity, $S \sim R_p v_{\rm esc}/v$, and the accretion rate can be approximated by $\dot{M} \sim \pi R_p v_{\rm esc}\tilde{\sigma}$. The scale height is roughly $h_d \sim v_z/\Omega$, where $v_z \sim I r\Omega$ is the *vertical* component of the dispersion velocity, with I being the characteristic orbital inclination of the disk particles. Accordingly, $S/h_d \sim R_p v_{\rm esc}\Omega/v v_z$. For a particle disk in collisional equilibrium, $I \sim e/2$, and v_z is comparable to v (e.g., Lissauer and Stewart 1993). The accretion then reduces to the usual expression, $\dot{M} \sim \pi R_p^2\sigma\Omega(v_{\rm esc}/v)^2$, with enhancement factor $F_g \sim (v_{\rm esc}/v)^2$. However, if the particle disk is *not* in equilibrium with $I \ll e$, then $v_z \ll v$, and the enhancement has an additional factor of $e/I \gg 1$. We will return to this possibility in the next section.

V. PROTOPLANET SURVIVAL

An irony that comes with an appreciation of protoplanet mobility is the worry that they could be too mobile for their own good. Comparison of the two rate expressions reveals that Type I exceeds Type II for masses greater than

$$\mu_o \sim \alpha\left(\frac{c_{\rm II}}{c_{\rm I}}\right)\left(\frac{M_\star}{\sigma r^2}\right)\left(\frac{c}{r\Omega}\right)^5 = \alpha\left(\frac{c_{\rm II}}{C_a}\right)\left(\frac{M_\star}{\sigma r^2}\right)\left(\frac{c}{r\Omega}\right)^4 \qquad (23)$$

This implies that such objects decay relative to disk material and could eventually be lost to the star. For example, if common values $\alpha \sim 10^{-3}$, $\sigma r^2/M_\star \sim 10^{-2}$, $c/r\Omega \sim 10^{-1}$, $c_{\rm II}/c_{\rm I} \sim 1$, are used, the nondimensional mass is $\mu_o \sim 10^{-6}$, which for a one-solar-mass primary corresponds to a fraction of an Earth mass ($\mu_\oplus = 3 \times 10^{-6}$). A longer lifetime can be enjoyed through Type II behavior, since the orbital decay is linked to the disk's evolution timescale.

A. The Threshold (Shiva) Mass

We expect Type I motions to be exhibited by small protoplanets and Type II motions exhibited by large protoplanets. A key question is, at what mass does a transition take place? Is it near $M_o = \mu_o M_\star$, where the rates are comparable, or does the transition threshold "overshoot" this value, so that there is a range of masses with orbital lifetimes *less* than the evolution timescale of the disk? Figure 1 shows a model calculation for protoplanet behavior at 5 AU in a minimum-mass solar nebula (Ward 1997*a*). The characteristic decay times are displayed as a function of mass measured in Earth masses. The curves are labeled by the strength of the turbulent viscosity. The transition from I to II occurs at a mass μ_S that depends on the specifics of the disk. In the limit of small viscosity, it approaches the inertial limit, μ_i [equation (15)]; in the high-viscosity limit, it approaches μ_ν [equation (14)]; but at intermediate values it is given approximately by (Ward 1997*b*)

$$\mu_S \approx c_S \alpha^{2/3} \left(\frac{M_\star}{\sigma r^2}\right)^{1/3} \left(\frac{c}{r\Omega}\right)^3 \tag{24}$$

where $c_S \sim O(1)$. This lies in the regime where Type I decay is up to two orders of magnitude faster than Type II. Thus, disk tides render the mass range ($\mu_o M_\star < M < \mu_S M_\star$) an especially precarious stage in the growth of a planet. [For this reason, the upper limit of this range has been named the "Shiva mass" after the Hindu god of destruction (Ward 1997*b*)]. For the range of α shown, planetary embryos between ~0.1 and 10 Earth masses are in danger of decaying out of the disk.

How does a planetary system survive this process? The characteristic growth time of an embryo that has run away in size from neighboring planetesimals is given by equation (15). Equating this to the Type I timescale gives us the protoplanet size that will decay out of the disk before significantly more growth can occur (Ward 1997*b*, 1998*b*):

$$\mu_{\rm crit} \approx \left(\frac{f_d F_g}{C_a}\right)^{3/4} \left(\frac{M_\star}{\rho_p r^3}\right)^{1/2} \left(\frac{c}{r\Omega}\right)^{3/2} \tag{25}$$

If $\mu_S < \mu_{\rm crit}$, the growing embryo can transition to Type II behavior before being lost from the system. The enhancement factor from stirring (Lissauer 1987; Ida and Makino 1993) is roughly, $F_g \sim F_{\rm stir} \approx 10^3(r/5\ \mathrm{AU})$. With this, equation (25) can be recast as

$$\frac{M_{\rm crit}}{\mathrm{M}_\oplus} \approx \left[\left(\frac{T}{150\ \mathrm{K}}\right)\left(\frac{1}{C_a}\right)\left(\frac{f_d}{0.01}\right)\left(\frac{F_g}{F_{\rm stir}}\right)\right]^{3/4} \tag{26}$$

and is independent of r except through the temperature gradient.

It is instructive to apply these concepts to our own solar system. If C_a is not too large, $M_{\rm crit}$ is comparable to an Earth mass, so that the terrestrial planets may have outlasted the nebula. For the outermost planets, it is even more likely that accretion was too slow for the critical mass to have been achieved during the disk's lifetime. However, neither of these alternatives are available for the giant planets. This seems to be a conundrum. They must form in the presence of the gas, and yet, the predicted decay time of their $\sim$10-$M_\oplus$ cores is much less than the generally assumed lifetime of the disk.

B. Nonequilibrium Accretion

In section IV.C, we discussed the accretion rate increase due to a decrease of planetesimal inclinations compared to eccentricities. We now want to consider how such a state could be produced. As a protoplanet migrates through the disk, it may encounter planetesimals with equilibrium (i.e., $I \sim e/2$) dispersion velocities lower than would be induced by stirring. The protoplanet will quickly force the eccentricities up, but, because its perturbations are primarily horizontal, the inclinations will increase more slowly from planetesimal interactions as the swarm seeks equipartition. If the protoplanet migrates rapidly enough compared to the relaxation time of the disk, the inclinations may be close to the pre-encounter values by the time they enter the protoplanet's feeding zone. Recent numerical experiments by Tanaka and Ida (1998) have confirmed this trend, as illustrated in Fig. 4. Their model calculations do reveal an enhanced accretion rate over the usual runaway prediction. Furthermore, the characteristic growth time is rather insensitive to protoplanet mass, in a manner similar to $\tau_{\rm swp}$. This implies that the ratio I/e must deviate more and more ($\propto 1/R$) from the equilibrium value as the protoplanet gets larger and quickens its migration. However, the efficiencies, $\mathcal{E} = \tau_{\rm swp}/\tau_{\rm growth}$ found by Tanaka and Ida (1998) tend to be low, i.e., $\lesssim$10%, so core formation appears possible only if the embryo migrates a distance considerably larger than its final orbit radius, and the nebula is at least a factor of 5 more massive than the minimum model. The possibility of planet loss weakens the rationale for the minimum-mass nebula model, so such conditions cannot be ruled out. On the other hand, these results are somewhat sensitive to the assumed starting inclinations and masses of the planetesimal swarm, and more study of this important issue is warranted.

VI. CONCLUSION

The discovery of extrasolar planets (see Marcy and Butler 1998) together with advances in our understanding of disk-planet interactions are leading to important revisions in the planetary formation paradigm. Before resonant interactions with the disk were considered, accretion models usually assumed that protoplanets spent most of their formative life in the general vicinity of their final orbits. The existence of close stellar companions

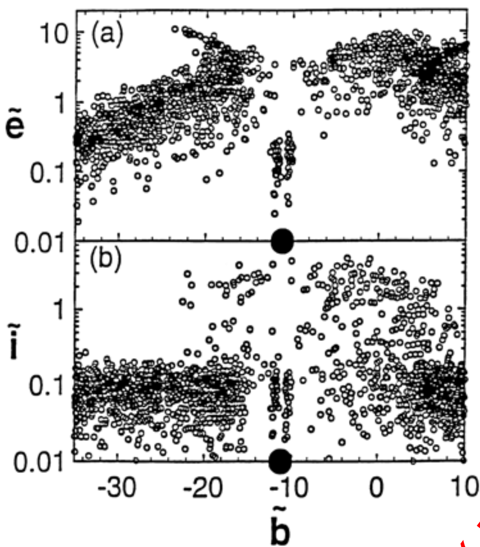

Figure 4. (a) Eccentricities $\tilde{e}$ and (b) inclinations $\tilde{i}$ of planetesimals normalized to $(\mu/3)^{1/3}$ produced by the stirring action of a migrating protoplanet. As the protoplanet approaches from the right, it pumps up their eccentricities while their inclinations are hardly enhanced from their initial values before they enter the feeding zone. Taken from Tanaka and Ida (1998).

provides persuasive evidence that this is not necessarily so. We have stressed that there are two distinct types of migrational behavior: Type I, wherein orbits decay relative to disk material due to an intrinsic imbalance of inner and outer disk torques, and Type II, wherein the protoplanet opens a gap and subsequently coevolves with the disk as it responds to its own angular momentum transport processes, such as viscous stresses.

Type II migration has been invoked by Lin et al. (1996) as the agent responsible for close stellar companions. Parking a planet just short of plunging it into the primary requires either a countertorque, such as stellar tides; a disk torque shutoff mechanism, such as a magnetospheric cavity around the star; or disk removal. Trilling et al. (1998) have presented detailed numerical models of Type II orbit decay, exploring the eventual outcomes for various star, planet, and disk combinations. Roche lobe overflow is a common event for gaseous planets not stabilized by stellar tides, during which the planet temporarily drifts away from the star. Although a providential dispersal of the nebula may save a very small fraction of these objects, most do not survive this process. It should be pointed out that the stabilizing mechanisms suggested by Lin et al. (1996) would also abort Type I orbit decay. Since these objects tend to be planets or cores that did not initiate gas accretion, Type I migration could result in a concentration of solids near the star. This may, in turn, be reflected in a high percentage of rock and CNO in the composition of close stellar companions (Ward 1997*b*).

Orbit migration due to disk torques may prevent embryos from becoming isolated, and thereby speed up their formation process, but it may also be an agent of destruction, driving newly formed protoplanets into their primaries. Thus, migration removes one motivation to assume a greater-than-minimum-mass nebula, but it supplies another by revealing a possible protoplanet loss mechanism that could decrease the overall accretion efficiency. Both migration types would eventually drive any planet into the primary if allowed to operate without restriction. Our own solar system is obvious evidence that planets can survive, but those recently discovered extrasolar planets occupying tight orbits suggest that it is sometimes a close call. Indeed, we cannot rule out a substantial mortality rate for newly formed planets. Understanding how these migration mechanisms are terminated is a crucial element for our comprehension of the planet-building process. At the very least, the mobility of large planetary objects introduces another degree of freedom in accretion modeling. In addition, disk torques provide a further damping mechanism for eccentricities and inclinations. This may affect dispersion velocities and alter the timescale and style of the accretion process. Although we do not yet understand all aspects of the interaction physics, even less is understood about their complex ramifications for cosmogonical models.

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